STRESSES & STRAINS

INTRODUCTION :-

The strength of material may be defined as the maximum resistance which a material can offer to the externally applied forces. The strength of materials is the study of the behaviour of structural & machine members under the action of external loads, taking in to account the internal forces created & the resulting deformations. Analysis is directed towards determining the limiting loads which the member can stand before failure of material or excessive deformation occurs.

STRESS

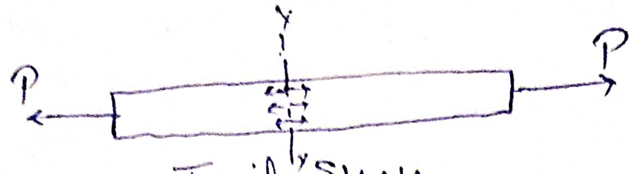
When some external forces are applied to a body, then body offers resistance to those forces. This internal resisting force per unit area is called 'stress'. The magnitude of resisting force is numerically equal to the applied forces externally. The internal resistance develops due to cohesive forces present between the molecules of body.

$$\text{Stress}(\sigma) = \frac{P \text{ (Load)}}{A \text{ (Area of cross-section)}} \quad \frac{N}{m^2} \text{ (unit)}$$

TYPES OF STRESSES

1. Tensile stress (σ_t) :-

When two equal & opposite forces acting in a line subjected to a body, such that they tends to increase or increase the length of body, then the stress develops due to these forces is known as tensile ~~force~~ stress. An example of tensile stress is provided by the rope attached to crane hook.



Tensile Stress
Fig 1.1

$$\sigma_t = \frac{P}{A} \sim \frac{N}{m^2}$$

Compressive Stress (σ_c)

When two equal & opposite forces acting in a line subjected to a body, such that they tends to decrease or decrease the length of body, then the stress develops due to these forces is known as compressive stress. An example of compressive stress is provided by leg of a table

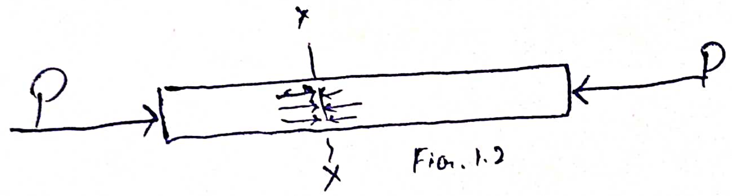
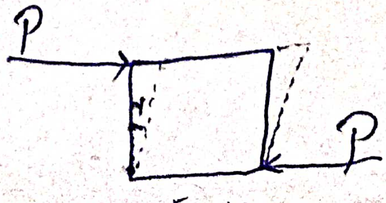


Fig. 1.2

$$\text{Compressive stress} (\sigma_c) = \frac{(P)\ \text{Load}}{(A)\ \text{Area of cross-section}} \qquad \sigma_c = \frac{P}{A} \sim \frac{N}{m^2}$$

Shear Stress (τ)

When two equal & opposite force acting tangentialy to a body, it tends to deform or deforms the body at a certain angle, the stress develops due to these forces is known as shear stress. An example of shear stress is provided by riveted joint.



$$\text{Shear Stress} (\tau) = \frac{P}{A} \sim \frac{N}{m^2}$$

Fig 1.3

Strain :-

Strain is a measure of the deformation produced in the member by the load.

$$\text{Strain}, (\epsilon) = \frac{\text{Change in length } (dl)}{\text{Original length } (l)}$$

, Unitless

TYPES OF STRAIN

Tensile Strain

It is the ratio of ~~decrease~~ increase in length to the original length.

$$\text{Tensile Strain}, \epsilon_t = \frac{\text{Increase in Length}}{\text{Original length}}$$

Compressive Strain

The ratio of decrease in length to the original length is known as compressive strain

$$\text{Compressive Strain } (\epsilon_c) = \frac{\text{Decrease in Length}}{\text{Original Length}}$$

Shear Strain :-

It is measure in terms of shear angle (ϕ).

$$\text{Shear angle} (\phi) = \frac{\text{Transverse Displacement}}{\text{Distance AD}} = \frac{dl}{h}$$

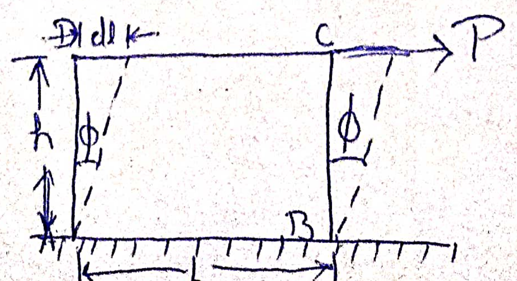


Fig 1.4

Volumeteric Strain :-

It is defined as the ratio between change in volume & original volume of the body.

$$\text{Volumeteric Strain}, E_v = \frac{\text{Change in Volume}}{\text{Original Volume}} = \frac{dV}{V}$$

HOOK'S LAW

It states that when a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. This means the ratio of the stress to corresponding strain is a constant with in elastic limit.

$$\text{Stress} \propto \text{Strain}$$

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

This constant is known as modulus of Elasticity or shear modulus.

MODULUS OF ELASTICITY OR YOUNG'S MODULUS

The ratio of tensile stress or compressive (Direct) stress to the corresponding strain is known as Young's modulus or Modulus of Elasticity & is denoted by E or Y.

$$E = \frac{\text{Direct Stress (Tensile or Compressive)}}{\text{Direct Strain}}$$

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$

MODULUS OF RIGIDITY OR SHEAR MODULUS

The ratio of the shear stress to corresponding shear strain with in elastic limit, is known as modulus of rigidity or shear modulus. This is denoted by C or G.

$$\text{Shear modulus, } G = \frac{\text{Shear stress } (\tau)}{\text{Shear strain } (\phi)}$$

BULK MODULUS

It is the ratio of direct stress (tensile or compressive) to the volumetric strain. It is denoted by K

$$\text{Bulk Modulus, } K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

STRESS STRAIN DIAGRAM

(a) FOR DUCTILE MATERIAL

When, we perform tensile test over a ductile material like mild steel & plot a curve between stress & strain, then the curve is known as stress-strain diagram for ductile material.

The test is carried out on a bar of uniform cross-section, usually circular, in a testing machine which indicates the tensile load being applied & corresponding strain develops. The following observations come through this curve.

Point a :

This limit is called proportional limit point. The stress corresponding to this point is called proportional limit stress (σ_a). It is denoted by σ_p. Up to this point, stress is directly proportional to strain. (Hook's law)

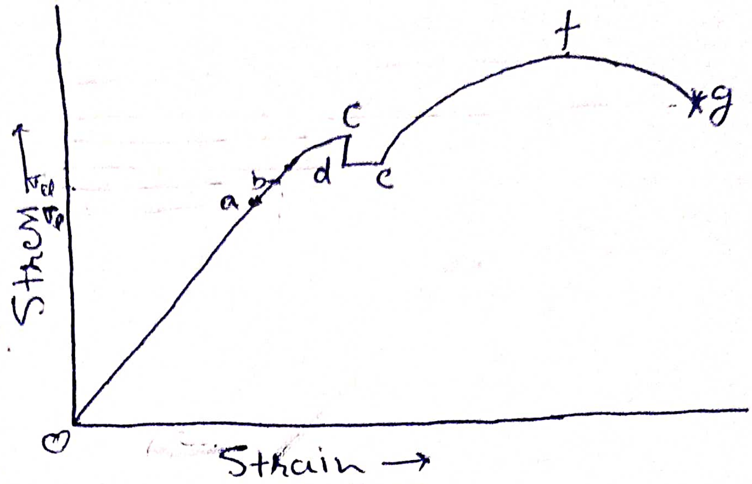


Fig 1.5

Point b :

This point is called elastic limit point. The stress corresponding to this point is called elastic limit stress. It is denoted by σ_{el}. This is a limiting value of force up to & within which, the deformation completely disappears on removal of force. ~~The value of stress corresponding to this~~

~~Point~~ Point C :

This point is called upper yield point. The stress corresponding to this point is called upper yield stress. It is denoted by σ_{uy}. If the material is stressed beyond the point b, plastic deformation will occur, i.e. strain which is not recoverable if the load is removed.

Point d & e :

These points are called lower yield point.

∴ The stress corresponding to these points are called lower yield stress. It is denoted by σ_{e}. From point c to d, the value of stress decreases. The portion de is called yielding of material at constant stress i.e. the specimen elongates by a considerable amount without any increase in stress from point e onwards, the strain hardening phenomenon becomes predominant & the strength of material increases, thereby, requiring more stress for more deformation, until point f is reached.

Point f :

Point f is called ultimate point & the stress corresponding to this point is called ultimate stress (σ_u). It is the maximum stress to which the material can be subjected in simple tensile test.
The strain in the region from c to f is in the region of 100 times that from O to C, & is partly elastic (i.e. recoverable), but mainly plastic (i.e. permanent strain).
At this stage (f) the bar begins to form a local "neck". Due to this local necking, the stress in the material goes on decreasing. ~~[illegible]~~

Point g :

Ultimately the specimen breaks at point g, known as breaking point & the corresponding

∴ stress is called the nominal breaking stress based upon the original area of cross-section.

(b) For Brittle materials

The stress-strain diagram for a brittle material like cast iron is as shown. There is very little elongation for such material.
Point a is limit of proportionality & g is breaking point.

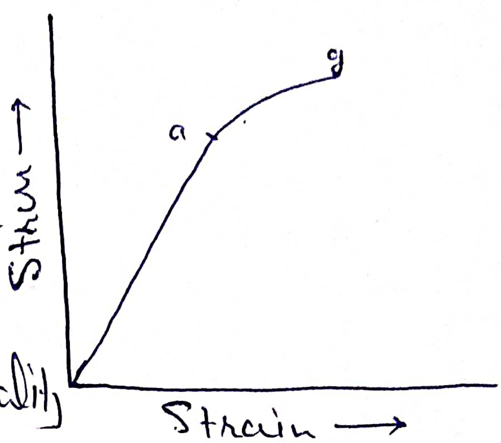


Fig 1.6

Problem 1.1

A rod 150 cm long & of diameter 2.0 cm is subjected to an axial pull of 20 kN. If the modulus of elasticity of the material of the rod is 2×10^5 N/mm²; Determine:
(i) the stress,
(ii) the strain, &
(iii) the elongation of the rod.

Sol. Given:

Length of rod, $l = 150$ cm $= 1500$ mm
Diameter of rod, $D = 2$ cm $= 20$ mm
∴ Area, $A = \frac{\pi}{4}(20)^2 = 100\pi$ mm²
Axial pull, $P = 20$ kN $= 20{,}000$ N
Modulus of Elasticity, $E = 2.0 \times 10^5$ N/mm²

(i) The stress $\sigma = \frac{P}{A} = \frac{20{,}000}{100\pi} = 63.66$ N/mm²

(ii) Strain $\varepsilon = \frac{\sigma}{E} = \frac{63.66}{2 \times 10^5} = 0.000318$

Deviation

(iii) Elongation is obtained by

$$e = \frac{dl}{L}$$

$$dl = e \times L$$

$$= 0.000318 \times 1500$$

$$= 0.0477 \times 10 =$$

$$= 0.477\,mm$$

Problem12 The following observations were made during a tensile test on a mild steel specimen 40mm in diameter & 200 mm long.

Elongation with 40 kN load with in proportional limit, δl = 0.0304mm

Yield load = 161 kN

Maximum load = 242 kN

Length of specimen at fracture = 249mm

Determine

(i) Young's modulus of elasticity

(ii) Yield point stress

(iii) Ultimate stress

(iv) Percentage elongation

Solution :- (i) Young's Modulus of Elasticity E

$$\text{Stress}, \ \sigma = \frac{P}{A} = \frac{40}{\frac{\pi}{4}(0.04)^2} = 3.18 \times 10^4 \ kN/m^2$$

$$\text{Strain}, \ \varepsilon = \frac{\delta l}{l} = \frac{0.0304}{200} = 0.000152$$

$$E = \frac{\sigma}{\varepsilon} = \frac{3.18 \times 10^4}{0.000152} = 2.09 \times 10^8 \ kN/m^2$$

(ii) Yield point stress = $\frac{\text{Yield point Load}}{\text{Area}}$

$$= \frac{161}{\frac{\pi}{4} \times (0.04)^2}$$

$$= 12.8 \times 10^4 \ KN/m^2$$

(iii) Ultimate stress:

Ultimate stress = $\frac{\text{Maximum Load}}{\text{Area}}$

$$= \frac{242}{\frac{\pi}{4} \times (0.04)^2}$$

$$= 19.2 \times 10^4 \ KN/m^2$$

(iv) Percentage elongation

%age elongation = $\frac{\text{Length of specimen at fracture} - \text{Original length}}{\text{Original length}}$

$$= \frac{249 - 200}{200} = 0.245$$

$$= 24.5\%.$$

Problem 1.3 :-

Find the minimum diameter of a steel wire, which is used to raise a load of 4000N if the stress not to exceed $95 MN/m^2$

Sol. Given

Load, $P = 4000N$

Stress, $\sigma = 95 \ MN/m^2 = 95 \times 106 \ N/m^2 = 95 N/mm^2$

Let D = Diameter of wire in mm

∴ Area of cross-section, $A = \frac{\pi}{4} D^2$

Now Stress, $\sigma = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$

$$95 = \frac{4000}{\frac{\pi}{4} \times D^2}$$

or $D^2 = \frac{4000 \times 4}{\pi \times 95}$

$\Rightarrow D = \sqrt{53.61}$

$= 7.32$ mm

PRINCIPLE OF SUPERPOSITION

When a number of loads are acting together on an elastic material, the principle of superposition states that the resultant strain will be the sum of individual strain caused by each load acting separately.

While using this principle for an elastic body which is subjected to a number of direct forces (tensile or compressive) at different section along the length of body, first the free body diagram of individual section is drawn. Then the deformation of each section is obtained. The total deformation of body will be then equal to the algebraic sum of deformations of the individual sections.

For example

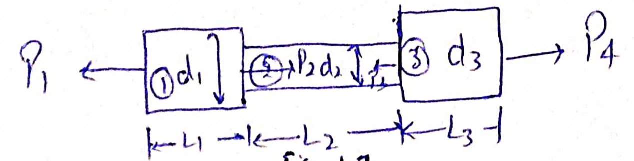


Fig. 1.7

Then its free body diagram

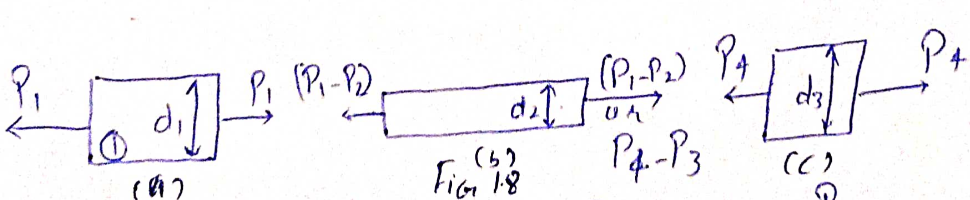


Fig 1.8

The stress for section ①, $\sigma_1 = \frac{P_1}{A_1}$

for section ②, $\sigma_2 = \frac{(P_1 - P_2) \text{ or } (P_4 - P_3)}{A_2}$

& $\sigma_3 = \frac{P_4}{A_3}$

Analysis Of Bars Of Varying Sections

A bar of different lengths & of different diameters is subjected to an axial load P.

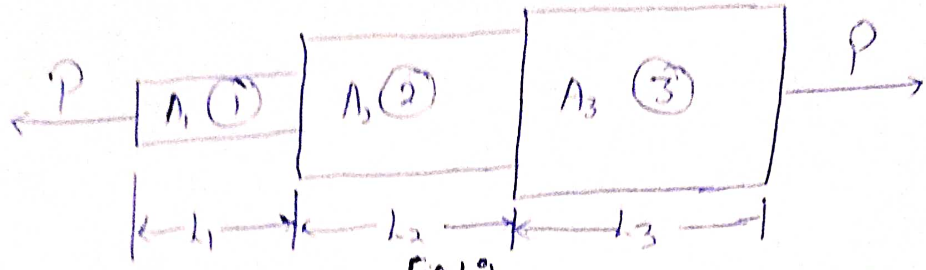


Fig 1.9

Let P = axial load acting on the bar

L_1 = Length of section (1),

A_1 = Cross-sectional area of section (1).

Similarly, L_2, A_2 = Length & cross-sectional area of section (2)

L_3, A_3 = Length & cross-sectional area of section (3)

& E = Young's Modulus of bar

Then stress for section (1)

$$\sigma_1 = \frac{P}{A_1}$$

Similarly for section (2) & (3)

$$\sigma_2 = \frac{P}{A_2} \quad \& \quad \sigma_3 = \frac{P}{A_3}$$

& Strain for section (1), $\epsilon_1 = \frac{\sigma_1}{E} = \frac{P}{A_1 E}$

Similarly for section (2) & (3)

$$\epsilon_2 = \frac{P}{A_2 E} \quad \& \quad \epsilon_3 = \frac{P}{A_3 E}$$

Now change in length of section (1), $dL_1 = e_1 L_1$

$$= \frac{P L_1}{A_1 E_1}$$

Similarly, $dl_2 = \frac{Pl_2}{A_2 E}$

& $dl_3 = \frac{Pl_3}{A_3 E}$

Total change in the length of the bar

$$dL = dL_1 + dL_2 + dL_3$$

$$= \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

$$\boxed{dL = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)}$$

If the sections are made up of different materials, then each section have different young's modulus i.e. E_1, E_2, E_3

$$\boxed{\therefore dL = P\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2} + \frac{L_3}{A_3E_3}\right]}$$

or in general.

We have, $\boxed{dL = P\sum_{i=1}^{n} \frac{L_i}{A_i E_i}}$

Problem 1.4 :- A round bar as shown in fig is subjected to a tensile load of 100 kN. What must be the diameter 'd' if the stress there is to be $100 MN/m^2$? Find also the total elongation. E = 290GPa

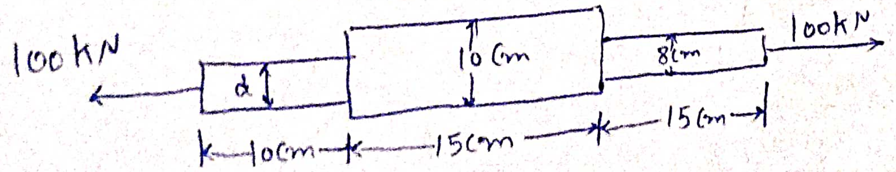


Fig 1.10

Sol. Stress, $\sigma = \frac{P}{A}$

Given $\sigma = 100 MN/m^2 = 100 \times 10^6 N/m^2$

$P = 100 kN = 100 \times 10^3 N$

& $A = \frac{\pi}{4} d^2$

$\therefore 100 \times 10^6 = \frac{100 \times 10^3}{\frac{\pi}{4} \times d^2}$

$\therefore$ Diameter, $d = \sqrt{\frac{4}{\pi \times 10^3}}$

$= 0.03568 m$

$= 35.68 mm$ Ans

Total elongation, $dL = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$

$= \frac{100 \times 10^3}{200 \times 10^9}\left[\frac{0.10}{10^3} + \frac{0.15}{\frac{\pi}{4} \times (100)^2} + \frac{0.15}{\frac{\pi}{4} \times (80)^2}\right] \times \frac{1}{10^6}$

$= 1.490 \times 10^{-4} = 0.0745 mm$ Ans

Problem 1.5: A steel bar 25 mm diameter is loaded as shown in fig. Determine the stress in each part & the total elongation. E = 210 GPa

Sol:-

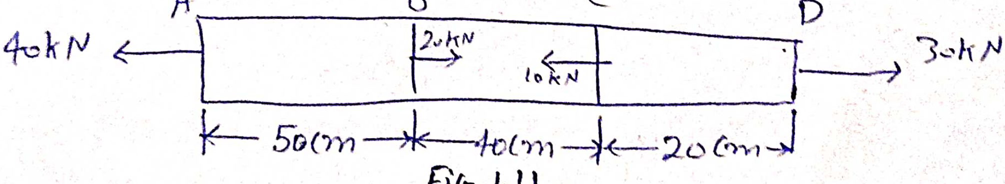


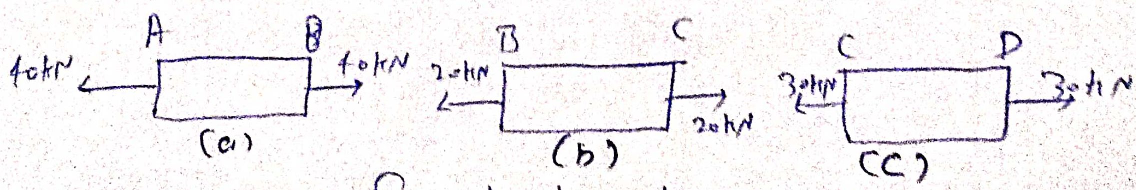
Fig 1.11

Sol. Given Diameter, $d = 25mm$

Area of cross-section $A = \frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times (25)^2 \times 10^{-6}$

$= 490.87 \times 10^{-6} m^2$



Free body diagrams

Fig 1.12

∴ Stress in various parts are.

$$\sigma_{AB} = \frac{40 \times 10^3}{490.87 \times 10^{-6}} = 81.488\ MN/m^2 \quad \underline{Ans}$$

$$\sigma_{BC} = \frac{20 \times 10^3}{490.87 \times 10^{-6}} = 40.744\ MN/m^2 \quad \underline{Ans}$$

$$\sigma_{CD} = \frac{30 \times 10^3}{490.87 \times 10^{-6}} = 61.116\ MN/m^2 \quad \underline{Ans}$$

Total elongation,

$$dL = \frac{1}{AE} \Sigma P_i L_i$$

$$= \frac{1}{490.87 \times 10^{-6} \times 210 \times 10^9} [40 \times 0.5 + 20 \times 0.4 + 30 \times 0.2] \times 10^3$$

$$= 0.3298\ mm \quad \underline{Ans}$$

~~Lateral Strain~~

Composite System

A system consisting of more than one bar or tube of same or different material rigidly connected in such a way that when subjected to loads or variation in temperature, each individual component undergoes equal changes in length is called a composite system.

ANALYSIS OF BARS OF COMPOSITE SECTION

A bar made up of two or more bars of equal lengths but of different materials

∴ Rigidly fixed with each other & behaving as one unit for extension or compression, when subjected to an axial tensile or compressive load in a composite bar.

For the composite bar the following two points are important.

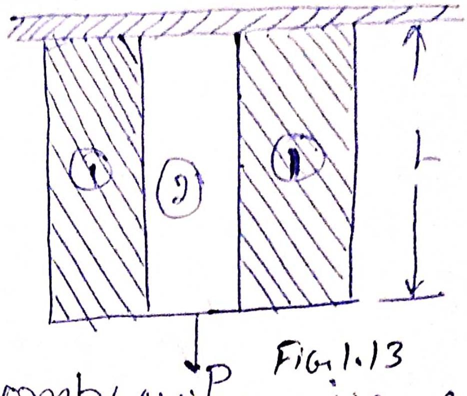


Fig 1.13

1. The extension or compression in each bar is equal. Hence the deformation per unit length i.e. strain in each bar is equal i.e.

$$\varepsilon_1 = \varepsilon_2$$

2. The total external load on the composite bar is equal to the sum of loads carried by each different material.

$$\boxed{P = P_1 + P_2}$$

Where, P = Total load on Composite bar

L = Length of composite bar

P_1 = Load shared by bar ① (or Tube ①)

P_2 = Load shared by bar ② (or Shaft ②)

<u>Problem 1.6</u> :- A steel rod of 3 cm diameter is enclosed centrally in hollow copper tube of external diameter 5 cm & internal diameter of 4 cm. The composite bar is then subjected to an axial pull of 45000 N. If the length of each bar is equal to 15 cm, determine:

(i) The stresses in the rod & tube, &
(ii) Load carried by each bar.
Take E for steel = $2.1 \times 10^5\ N/mm^2$ & for copper = $1.1 \times 10^5\ N/mm^2$

Sol. Given:

Diameter of steel rod = 3cm
, d_s = 30 mm

∴ Area of steel rod,

$$A_s = \frac{\pi}{4}(30)^2$$
$$= 706.86\ mm^2$$

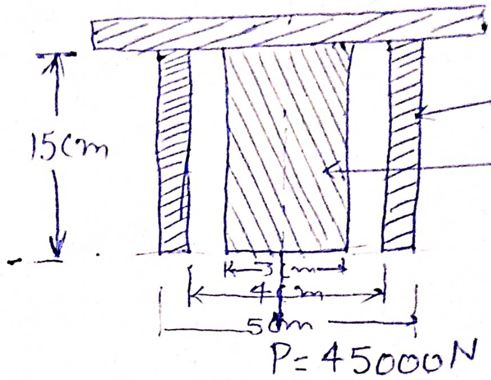


Fig 1.14

External diameter of copper tube, d_o = 5cm = 50mm
Internal diameter of copper tube, d_i = 4cm = 40mm
Area of copper tube (Area of cross-section) $A_c = \frac{\pi}{4}(d_o^2 - d_i^2)$

$$\Rightarrow A_c = \frac{\pi}{4}[50^2 - 40^2] = 706.86\ mm^2$$

Axial pull, P = 4500 N
Length, L = 15 cm
Young's modulus for steel, $E_s = 2.1 \times 10^5\ N/mm^2$
Young's Modulus for copper, $E_c = 1.1 \times 10^5\ N/mm^2$

(i) The stress in the rod & tube

Let σ_s = Stress in steel
P_s = Load carried by steel rod
σ_c = Stress in copper
P_c = Load carried by copper tube

Now, Strain in steel = Strain in copper

$$\varepsilon_s = \varepsilon_c$$

or $$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\therefore \sigma_s = \frac{E_s}{E_c} \times \sigma_c$$

$$= \frac{2.1 \times 10^5}{1.1 \times 10^5} \times \sigma_c$$

$$= 1.909\, \sigma_c \quad \text{--- (1)}$$

Now Total load $P = P_s + P_c$

or $45000 = \sigma_s \times A_s + \sigma_c \times A_c$ $\left\{\because \sigma_s = \frac{P_s}{A_s} \text{ \& } \sigma_c = \frac{P_c}{A_c}\right\}$

or $45000 = 1.909\sigma_c\ 706.86 + \sigma_c \times 706.86$ $\{\because$ from (1)$\}$

or $45000 = \sigma_c\,(1.909 \times 706.86 + 706.86)$

or $\sigma_c = \frac{45000}{2056.26} = 21.88\ N/mm^2$ Ans

By putting value of σ_c in eqn. (1)

$$\sigma_s = 1.909 \times 21.88$$

$= 41.77\ N/mm^2$ Ans

(ii) Load carried by each bar

Load carried by steel rod

$$P_s = \sigma_s \times A_s = 41.77 \times 706.86$$

$$= 29525.5\ N$$

Load carried by copper tube

$$P_c = \sigma_c \times A_c = 21.88 \times 706.86$$

$= 15474.5\ N$ Ans

Problem 1.7 - A steel rod 20mm diameter is passed through a brass tube 25mm internal diameter & 30mm external diameter. The tube is 80cm long & is closed by thin rigid washers & fastened by nuts, screwed to the rod. The nuts are tightened until the compressive force in the tube is 5KN. Calculate the stresses in the rod & in the tube.

$E_s = 200\ GPa$, $E_b = 80\ GPa$

Sol.

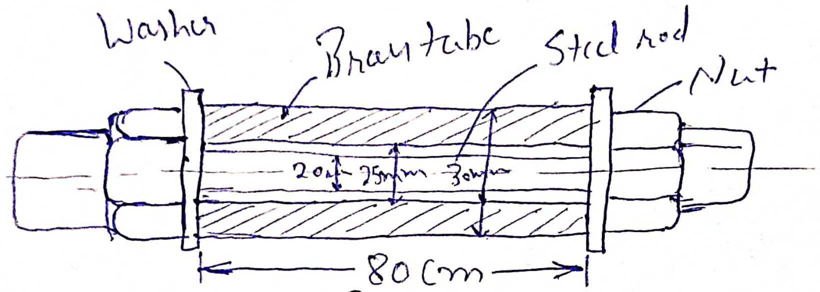


FIG 1.15

Since the rod & tube are rigidly fixed, therefore, strains in both are same.

Area of steel rod, $A_s = \frac{\pi}{4} \times (20)^2 = 314\ mm^2$

Area of Brass tube, $A_b = \frac{\pi}{4}(30^2 - 25^2) = 216\ mm^2$

Let dL = decrease in length in mm

$\therefore$ Strain in tube & rod $\varepsilon_s = \varepsilon_b = \frac{dL}{L} = \frac{dL}{800}$

Stress in rod, $\sigma_s = \frac{\Delta L}{800} \times E_s$

$= \frac{\Delta L}{800} \times 200 \times 10^9 = \frac{\Delta l \times 10^9}{4}\ Pa$

Stress in tube, $\sigma_b = \frac{\Delta l}{800} \times E_b = \frac{\Delta l}{800_{10}} \times \cancel{80} \times 10^9$

$$\sigma_b = \Delta L \times 10^8 \text{ Pa}$$

Load or force in rod, $P_s = \sigma_s \times A_s$

$$= \frac{\Delta L}{4} \times 10^9 \times 314 = 78.5 \times \Delta L \times 10^3 \text{ N}$$

Load or force in tube, $P_b = \sigma_b \times A_b$

$$= \Delta L \times 10^8 \times 216 = 21.6 \times \Delta L \times 10^3 \text{ N}$$

Total force $P = P_s + P_b$

$$\therefore \Delta L (78.5 + 21.6) \times 10^3 = 5 \times 10^3$$

$$\Delta L = \frac{5}{100.1} = 0.0499 \text{ mm} \quad \underline{\text{Ans}}$$

$$\therefore \sigma_s = \frac{0.0499}{4} \times 10^3 = 12.48 \text{ MPa } (\text{Tensile}) \text{ Ans}$$

$$\sigma_b = 0.0499 \times 10^2 = 4.99 \text{ MPa} \quad \underline{\text{Ans}}$$

<u>Lateral Strain</u>:- The strain at the right angle to the direction of applied load is known as lateral strain.

Let a rectangular bar of length L, breadth b & depth d is subjected to an axial load P

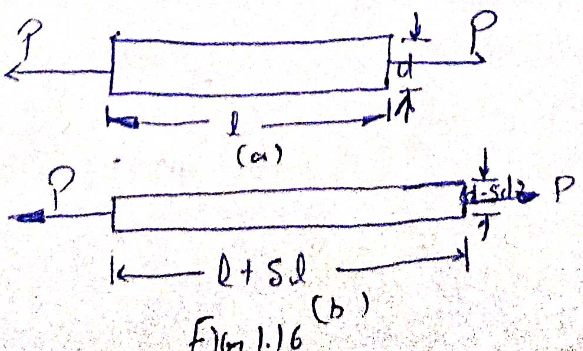


Fig 1.16

∴ The length of bar will increase while breadth & depth will decrease.

Let δl = Increase in length

δb = Decrease in breadth &

δd = Decrease in depth

Then, Longitudinal strain $= \frac{\delta l}{l}$

& Lateral strain $= \frac{\delta b}{b}$ or $\frac{\delta d}{d}$

<u>Poisson's Ratio</u>:- The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within elastic limit. This ratio is called Poisson's ratio. It is also defined as the negative ratio of strain in another direction to strain in loading direction. It is denoted by

ν or $\frac{1}{m}$

$$\therefore \boxed{\text{Poisson ratio} \left(\nu \text{ or } \frac{1}{m}\right) = -\frac{\text{Lateral strain}}{\text{Longitudinal strain}}}$$

<u>Elastic Constants & their relationships</u>

1. E - Young's Modulus of Elasticity $= \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$
2. G - Modulus of rigidity or shear modulus $= \frac{\text{Shear stress}}{\text{Shear strain}}$
3. K - Bulk Modulus $= \frac{\text{Direct stress}}{\text{Volumetric strain}}$
4. ν - Poisson ratio $= -\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

∴ Relationship between them

$$\boxed{E = 2G(1+\nu) = 3K(1-2\nu)}$$

~~Rel~~ <u>Relationship between E & K</u> (Derivation)

Consider a cube ABCDEFGH which is subjected to three mutually perpendicular tensile stresses of equal intensity.

Let

L = length of cube

dL = Change in length of cube

E = Young's modulus of material

ν = Poisson's ratio

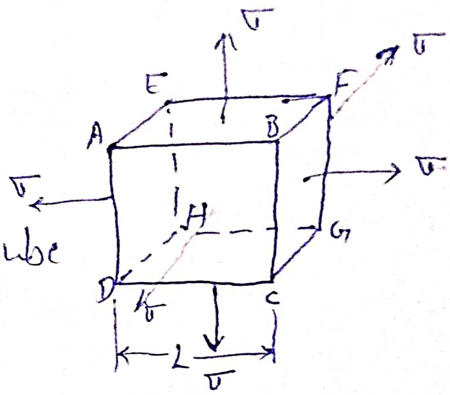


FIG 1.17

∴ Volume of cube, $V = L^3$

Now, let us consider the strain of one of sides of the cube (say AB) under the action of three mutually perpendicular stresses.

1. Strain of AB due to elongation, $\varepsilon_1 = \frac{\sigma}{E}$
2. Strain of AB due to compression of BF, ε_2

i.e. Lateral strain = − Poisson's ratio × Longitudinal strain

$$= -\nu \times \frac{\sigma}{E}$$

$$\therefore \varepsilon_2 = -\frac{\nu\sigma}{E}$$

Similarly, due to BC, $\varepsilon_3 = -\frac{\nu\sigma}{E}$

Total strain of AB is given by

$$\frac{dL}{L} = \frac{\sigma}{E} - \frac{\nu\sigma}{E} - \frac{\nu\sigma}{E}$$

$$= \frac{\sigma}{E}(1-2\nu) \quad (1)$$

Now, Volume of cube, $V = L^3$

Differentiate w.r.t. L

$$\frac{dV}{dL} = 3L^2$$

or $dV = 3L^2 dL$

Divide both sides by V

$$\frac{dV}{V} = \frac{3L^2 dL}{V}$$

$$\Rightarrow \frac{dV}{V} = \frac{3\cancel{L^2} dL}{L^{\cancel{3}}}$$

$$\Rightarrow \frac{dV}{V} = 3\frac{dL}{L} \quad (2)$$

By putting the value of $\frac{dL}{L}$ from eqn. (1)

$$\therefore \frac{dV}{V} = 3\frac{\sigma}{E}(1-2\nu)$$

$$\Rightarrow E = 3\frac{\sigma}{\frac{dV}{V}}(1-2\nu)$$

$$\Rightarrow \boxed{E = 3K(1-2\nu)} \quad \left\{\because K = \frac{\sigma}{\frac{dV}{V}}\right\}$$

(ii) <u>Relationship between E & G (Derivation)</u>

Let us consider a square block ABCD of unit thickness is subjected to a set of shear stresses of intensity τ on the faces AB and CD. Then,

→ Complimentary shear stresses of same magnitude is set up on faces AD & BC

The block distorts to new configuration $ABC'D'$. The diagonal AC elongates will experience a tensile stress of magnitude τ. & diagonal BD will experience a compressive stress of magnitude τ.

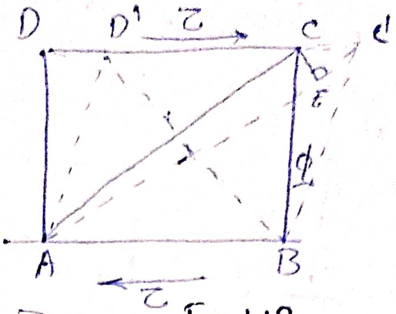


Fig 1.18

Now, Increase in length of diagonal $AC = AC' - AC$

$\therefore$ Tensile strain in the diagonal AC

$$= \frac{\text{Increase in length}}{\text{Original length}}$$

$$= \frac{AC' - AC}{AC}$$

From C, draw a perpendicular CE on AC'.

Since CC' extension is small, & hence angle $\angle CAC'$ will be very small. Hence we can take

$$AC = AE$$

$$\& \angle AC'B = \angle ACB = 45^\circ$$

$$\therefore \text{ Strain in diagonal } AC = \frac{AC' - AE}{AC} = \frac{EC'}{AC} \quad -(1)$$

& In $\triangle CEC'$

$$\frac{EC'}{CC'} = \cos 45^\circ$$

$$EC' = CC' \times \frac{1}{\sqrt{2}}$$

By putting this in eqn. (1)

$$\therefore \text{ Strain in diagonal } AC = \frac{CC'}{\sqrt{2}\,AC} = \frac{CC'}{\sqrt{2}\,\sqrt{2}\,BC}$$

$$\left\{ \because AC = \sqrt{AB^2 + BC^2} = \sqrt{2BC^2} = \sqrt{2}\,BC \right\}$$

Strain in diagonal AC $= \frac{CC'}{2BC} = \frac{\tan\phi}{2}$ $\{\because \tan\phi = \frac{CC'}{BC}$

$= \frac{\phi}{2}$ — (2)

$\because$ ϕ is small

$\therefore$ $\tan\phi \simeq \phi$

ϕ represents the shear strain

In terms of shear stress τ & modulus of rigidity

Shear strain $= \frac{\tau}{G}$

$\therefore$ Strain in diagonal AC $= \frac{\tau}{2G}$ — (3) $\{\because$ from (2)$\}$

Now, Strain in diagonal AC is also given by

= Strain due to tensile stress in AC − Strain due to compressive stress in BD

Strain due to tensile stress in AC $= \frac{\tau}{E}$

Strain due to compressive stress in BD (Lateral strain)

$= -\nu\frac{\tau}{E}$

$\therefore$ Strain in diagonal AC $= \frac{\tau}{E} - \left(-\nu\frac{\tau}{E}\right)$

$= \frac{\tau}{E}(1+\nu)$ — (4)

By comparing (3) & (4)

$\therefore$ $\frac{\tau}{2G} = \frac{\tau}{E}(1+\nu)$

$\Rightarrow$ $\boxed{E = 2G(1+\nu)}$

(iii) Relation between E, G & K

$$E = 2G(1+\nu) = 3K(1-2\nu)$$

To eliminate ν from these expression in terms of E

$$\nu = \frac{E}{2G} - 1$$

$$E = 3K\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]$$

or $$E = 3K\left[1 - \left(\frac{E}{G} - 2\right)\right]$$

$$= 3K\left[3 - \frac{E}{G}\right]$$

$$= 9K - \frac{3KE}{G}$$

or $$E + \frac{3KE}{G} = 9K$$

or $$\frac{EG + 3KE}{G} = 9K$$

or $$E\left(\frac{G+3K}{G}\right) = 9K$$

or $$E = \frac{9KG}{G+3K}$$

$$\boxed{\therefore E = 2G(1+\nu) = 3K(1-2\nu) = \frac{9KG}{G+3K}}$$

Problem 1.8 :- A steel rod 5m long & 30 mm in diameter is subjected to an axial tensile load of 50 kN. Determine the change in length & diameter of rod. Take $E = 2\times10^5\ N/mm^2$ & $\nu = 0.25$

Sol. Given:

Length, $l = 5m = 5000mm$, Load $P = 50\times10^3\ N$

Diameter, $d = 30\ mm$

$E = 2\times10^5\ N/mm^2$

$\nu = 0.25$

Let δd = Change in diameter

δl = Change in length

Now, strain of length $= \dfrac{\text{Stress}}{E}$

$$= \frac{P}{\frac{\pi}{4}d^2} \times \frac{1}{E}$$

$$= \frac{50\times10^3}{\frac{\pi}{4}\times(30)^2} \times \frac{1}{2\times10^5}$$

$$= 0.00035$$

Also, strain $= \dfrac{\delta L}{L}$

$\therefore \dfrac{\delta L}{L} = 0.00035$

$\Rightarrow \delta L = 0.00035 \times 5\times10^3$

$= 1.768\ mm$ Ans

Now, lateral strain = $\nu \times$ Longitudinal strain

$= 0.25 \times 0.00035$

$= 0.0000884$

Also, Lateral strain $= \frac{\delta d}{d}$

$\therefore \frac{\delta d}{d} = 0.0000884$

$\Rightarrow \delta d = 0.0000884 \times 30$

$= 0.00265$ mm Ans

Problem 1.9 - A bar of 20mm diameter is subjected to a pull of 50KN. The measured extension over a gauge length of 20 cm is 0.1mm & the change in diameter is 0.0035mm. Calculate the poisson's ratio & modulus of Elasticity E & Bulk modulus K.

Sol. Area of cross-section of Bar,

$$A = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4} \times (20 \times 10^{-3})^2 = 314.16 \times 10^{-6} m^2$$

Longitudinal strain

$$\varepsilon_l = \frac{\delta l}{l}$$

$$= \frac{0.1}{20 \times 10} = 5 \times 10^{-4}$$

Lateral strain $= \frac{\delta d}{d}$

$$= \frac{0.0035}{20} = 1.75 \times 10^{-4}$$

ν, Poisson's ratio $= \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$$\nu = \frac{1.75 \times 10^{-4}}{5 \times 10^{-4}} = 0.35$$

Modulus of Elasticity,

$$E = \frac{P l}{A \delta l}$$

$$= \frac{50 \times 10^3 \times 20 \times 10^{-2}}{314.16 \times 10^{-6} \times 0.1 \times 10^{-3}}$$

$$= 318.31 \text{ GN/m}^2 \quad \underline{\text{Ans}}$$

Bulk Modulus

$$E = 3K(1-2\nu)$$

or $$K = \frac{E}{3(1-2\nu)}$$

$$= \frac{318.31}{3(1-2 \times 0.35)}$$

$$= 353.68 \text{ GN/m}^2 \quad \underline{\text{Ans}}$$

HIGHLIGHTS

1. The internal resistance developed per unit area is called 'stress'. The internal resistance develops due to cohesive forces between molecules of body.

$$\text{Stress } (\sigma) = \frac{P \text{ (LOAD)}}{A \text{ (cross-sectional area)}}$$

2. Stress is expressed as kgf/m^2, kgf/cm^2, N/m^2 & N/mm^2.

3. The tensile stress develops in a body, when it is subjected with two equal & opposite forces, such that they tends to increase or increases the length of body;

4. When two equal & opposite forces acting in a line subjected to a body, such that they tends to decrease or decrease the length of body along the axis of applied force, then stress is said to be compressive stress.

5. The ratio of ~~change~~ change in dimension of the body to the original dimension is known as strain.

6. Hook's law states that within elastic limit, the stress is directly proportional to strain.

7. The ratio of tensile stress or compressive stress to the corresponding strain is known as Young's Modulus or Modulus of Elasticity (E)

$$E = \frac{\text{Tensile or Compressive Stress}}{\text{Corresponding Strain}}$$

8. The ratio of shear stress to corresponding shear strain within elastic limit, is known as Modulus of rigidity or shear Modulus (G)

$$G = \frac{\tau \text{ (Shear stress)}}{\phi \text{ (Shear strain)}}$$

9. The ratio of direct stress to the volumetric strain is known as Bulk modulus (k).

$$K = \frac{\text{(Direct stress) } \sigma}{\text{Volumetric strain } \frac{dV}{V}}$$

10. The curve between stress & strain for a ductile or brittle material is known as stress-strain diagram.

11. When a number of loads are acting together on an elastic material, the principle of superposition states that the resultant strain ~~caused by each load acting~~ will be sum of individual strain caused by each load acting separately.

(12). Total change in the length of a bar of different lengths & of different diameters when subjected to an axial load P, is given by

$$dL = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} + \cdots\right] \text{ when E is same}$$

$$= P\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2} + \frac{L_3}{A_3E_3} + \cdots\right] \text{ when E is different}$$

13. In case of composite bar made up of two or more bars of equal lengths but of different material rigidly fixed with each other: (i) Strain in each bar is equal

$$\varepsilon_1 = \varepsilon_2$$

(ii) The total external load on the composite bar is equal to the sum of loads carried by each bar

$$P = P_1 + P_2$$

14. Poisson's ratio is the negative ratio of lateral strain to the longitudinal strain.

15. The relationships between elastic constants

$$E = 2G(1+\nu) = 3K(1-2\nu) = \frac{9KG}{G+3K}$$

EXERCISE 1

(A) Theoretical Questions

1. Define stress & strain & their types.

2. Discuss the stress-strain diagram for a ductile material. How does it differ from the one for brittle material?

3. State the following:
   Hooke's law,
   Elastic limit,
   Upper Yield point,
   Lower Yield point,
   Ultimate stress,
   Breaking stress,
   Working stress,
   Factor of Safety

4. Three sections of a bar are having different lengths & different diameters. The bar is subjected to an axial load P. Determine the total change in length of the bar. Take Young's Modulus of different sections same.

5. Define a composite bar. How will you find the stresses & load carried by each member of a composite bar?

6. Define Poisson's ratio & Explain clearly the difference between lateral strain & longitudinal strain.

7. What is a bulk modulus? Derive an expression for Young's Modulus in terms of bulk modulus & Poisson's ratio.

8. Derive the following relations for the elastic constants for an isotropic material:

(i) $E = 2G(1+\nu) = 3K(1-2\nu)$

(ii) $E = \dfrac{9KG}{3K+G}$

(B) NUMERICAL PROBLEMS

1. A mild steel rod 20 mm diameter is subjected to an axial pull of 50 kN. Determine the tensile stress induced in the rod & the elongation if the unloaded length is 5m. $E = 210\,GN/m^2$

[Ans:- $\sigma = 159.155\,MN/m^2$
$\delta = 3.789$ mm]

2. During a tensile test on a mild steel specimen, 400 mm diameter & 200 mm long, the following data was obtained:
Extension at 40 kN load = 0.0304 mm
Yield load = 161 kN & length of specimen at fracture = 249 mm
Determine (a) Modulus of elasticity
(b) Percentage elongation
(c) Yield point stress

[Ans
$E = 2.05 \times 10^5\,N/mm^2$
$\% \delta L = 24.5\%$
Yield point stress $\sigma_{y} = 128.18\,N/mm^2$]

3. A brass bar, having cross-sectional area of 1000mm², is subjected to axial force as shown in fig 1.19

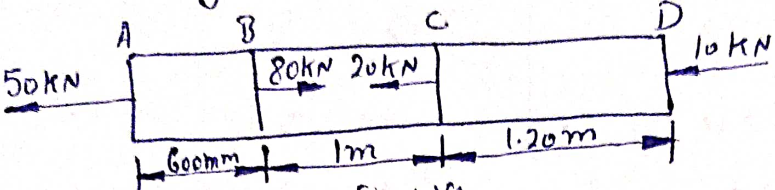


Fig 1.19

Find the total elongation of bar.
Take $E = 1.05 \times 10^5 \ N/mm^2$

[Ans: -0.11 mm]

4. An axial pull of 35000N is acting on a bar consisting of three lengths as shown in fig 1.20

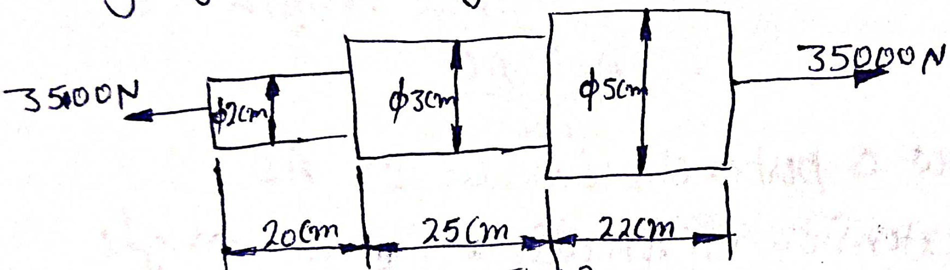


Fig 1.20

If $E = 2.1 \times 10^5 \ N/mm^2$, Determine
(i) Stress in each section
(ii) Total extension of the bar.

[Ans:- $\sigma_1 = 111.408 \ N/mm^2$, $\sigma_2 = 49.51 \ N/mm^2$, $\sigma_3 = 17.82 \ N/mm^2$]

5. A steel bar of 30mm diameter is loaded as shown in fig 1.21. Determine the stress in each portion & the total elongation. $E = 210$ GPa

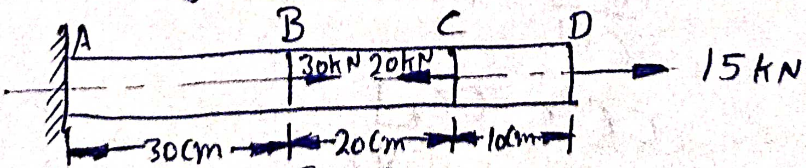


Fig 1.21

Ans. ~~31.4159 kN~~, $\sigma_{AB} = 35.36$ MPa

$\sigma_{BC} = -7.073$ MPa

$\sigma_{CD} = 21.221$ MPa

$dL = 0.0539$ mm

6. A brass tube of 60 mm outside diameter completely encloses a steel bar of 40 mm diameter. The composite system measures 300 mm in length & carries an axial thrust which induces a thrust equal to $50\ N/mm^2$ in the brass tube. Make calculations for the

(a) Stress developed in steel bar & ~~[illegible]~~ tube

(b) Change in length of composite bar

Take $E_s = 210\ GPa$

$E_b = 10^5\ GPa$

[Ans $100\ N/mm^2$, $0.1428\ mm$]

7. A reinforced short concrete column 250 mm × 250 mm in section is reinforced with 8 steel bars. The total area of steel bars is $2500\ mm^2$. The column carries a load of 390 kN. If the modulus of elasticity for steel is 15 times that of concrete, find the stresses in concrete & steel.

[Ans $\sigma_c = 4\ N/mm^2$, $\sigma_s = 60\ N/mm^2$]

8. A steel bar 40 mm × 40 mm × 300 cm long is subjected to an axial pull of 12.8 kN. Calculate change ~~in~~ in length & breadth of the bar.

Take $E = 200\ GPa$, $\nu = 0.3$

[Ans, dL = ~~[illegible]~~ 1.2 mm, $\delta b = -0.0048\ mm$]

9. A cylindrical bar is 2 cm in diameter & 100 cm long. During a tensile test it is found that the longitudinal strain is 4 times the lateral strain. Calculate the modulus of rigidity & the bulk modulus if its elastic modulus is 100 GPa.

[Ans $G = 40\ GPa$, $K = 66.67\ GPa$